

Extinction of diffusion flames with nonunity Lewis numbers

J. S. KIM* and F. A. WILLIAMS

*Center for Energy and Combustion Research, Department of Applied Mechanics and Engineering Sciences,
University of California, San Diego, La Jolla, CA 92093-0411 U.S.A.*

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Abstract. Quasisteady extinction of diffusion flames with nonunity Lewis numbers is analyzed for counterflow diffusion flames in the diffusion-flame regime of activation-energy asymptotics. Particular attention is placed on an excess or deficiency of the total energy in the reaction region, associated with leakage of the reactants. If the Lewis number is less than unity, there is diminished diffusive loss of thermal energy that leads to an increase of the total energy in the reaction zone as reactants penetrate. The resulting excess total energy strengthens the chemical reaction, so that the flame becomes more robust and resistant to extinction. On the other hand, flames with Lewis numbers greater than unity are found to extinguish more easily. An extinction criterion is provided that is valid for nonunity Lewis numbers.

Key words: diffusion flame, Lewis number, Damköhler number, extinction, excess enthalpy

1. Introduction

Analysis of diffusion-flame structures has been greatly facilitated by the assumption of infinite-rate chemistry [1]. Under this assumption, the chemistry is reduced to a set of algebraic equations describing chemical equilibrium. In particular, if an overall one-step irreversible reaction is adopted, the infinite-rate chemistry corresponds to the Burke-Schumann limit, in which the fuel and oxidizer streams are separated by an infinitely thin reaction sheet located where stoichiometric consumption of fuel and oxidizer is achieved. The Burke-Schumann limit has met with some success in providing satisfactory first approximations for flame temperature, reaction-zone location, fuel consumption and other quantities that are mainly controlled by transport processes. However, the Burke-Schumann limit is unable to predict phenomena associated with finite-rate chemistry, such as pollutant production, leakage of reactants and, in particular, extinction. Prediction of these phenomena requires integration of a set of partial differential equations with highly nonlinear chemical-reaction terms.

For simplified chemical models employing an overall one-step irreversible Arrhenius-type reaction rate with a large Zel'dovich number (a measure of the ratio of the activation energy to the thermal energy), activation-energy asymptotics (AEA) has been developed, enabling integration of the reaction rate over the thin reaction zone, thinner by an order of magnitude than the thick transport zones. For diffusion flames, Liñán [2] derived a standard AEA procedure to analyze diffusion-flame structures, providing an extinction criterion readily used to find an extinction condition if only the Burke-Schumann solution is known. The AEA extinction criterion was later applied to find overall activation energies and frequency factors by a comparison with the results of the extinction experiments [3, 4]. The AEA procedure still remains one of the methods favored to construct flamelet data used in turbulent diffusion-

* Present Address: Environment Research Center, Korea Institute of Science and Technology, PO Box 131, Cheongryang, Seoul, 130-650, Korea

flame models [5, 6]. Although the extinction criterion proposed by Liñán was derived only for flames with Lewis numbers of unity, this extinction criterion often has been applied to flames with nonunity Lewis number without modifications to take into account all effects coming from nonunity Lewis number [7, 8]. The full analysis valid for nonunity Lewis numbers still has not been published. It is the purpose of the present paper to explain these differences by demonstrating an AEA analysis that is generalized for nonunity Lewis numbers.

For diffusion flames with nonunity Lewis number, the usual conserved scalar [1] does not exist, so that it is more difficult to decouple the temperature variation from the reaction term in species conservation. Although generalized coupling functions are defined by Liñán [9], their distributions are still dependent on reactant leakages caused by finite-rate chemistry. Among the generalized coupling functions, particular attention needs to be paid to the total enthalpy, which can be constructed by the addition of a suitable combination of the species-conservation equations to the thermal-energy-conservation equation to eliminate the reaction term. If the Lewis number is unity, the total enthalpy is a conserved scalar, in that its profile does not vary, even in the presence of finite-rate chemistry. However, if the Lewis number departs from unity, as finite-rate chemistry causes the reactants to leak through the reaction sheet, the total enthalpy may decrease or increase because of the unbalance of diffusion for chemical and thermal energies. Although the variation of the total enthalpy remains small (typically of the order of the reciprocal of the Zel'dovich number), each small variations are capable of leading to a change of the reaction rate by a relative amount of order unity because of the large Zel'dovich number. In the previous AEA analyses, that were aimed at describing extinction of diffusion flames with nonunity Lewis number, the effects of the total-enthalpy variation with reactant leakage were not properly taken into account, so that inaccuracies of order unity can arise in their extinction results. In addition, some of the oscillatory and cellular structures of near-extinction flames are found to result from the variation of the total enthalpy [10, 11, 12].

Since we cannot cover all the phenomena associated with nonunity Lewis number, our scope in this paper will be restricted to the phenomenon of diffusion-flame extinction. Oscillatory and cellular instabilities of diffusion flames will be discussed in forthcoming publications. The extinction analysis will mostly be parallel to that of Liñán. Therefore, emphasis will be placed on how to include the effects of the total-enthalpy variation and what their consequences might be. Each algebraic step of the analysis will be shown in detail to provide readers with a pedagogic background. In addition, for the sake of clarity, many simplifying assumptions will be made to reduce clutter in the equations. For example, Lewis numbers for both reactants will be taken as constant and equal. Further calculations therefore need to be completed before the full range of applications can be addressed.

2. Conservation equations and boundary conditions

Since our present purpose is a demonstration of an extinction analysis for diffusion flames with nonunity Lewis numbers, instead of accurately predicting flame structures in general situations, a number of assumptions are introduced for simplicity. The flame configuration employed in this analysis is a counterflow diffusion flame as illustrated in Figure 1. The density in the flow is assumed to be constant, because variable density is not an essential generalization to illustrate the elements of the analysis. All molecular properties are also assumed to be constant. To simplify the molecular diffusion mechanism, the two reactants, fuel and oxidizer, are assumed to be present in small amounts in an abundance of inert gas. In such a case, the diffusion coefficients of fuel and oxidizer are their binary diffusion coefficients

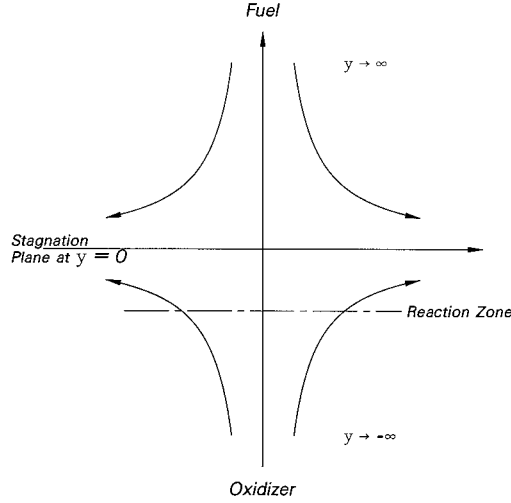


Figure 1. Schematic diagram of a counterflow diffusion flame.

with respect to the inert, and the thermal diffusion coefficient D_T^* of the system is given by that of the inert. The superscript * denotes dimensional quantities. In addition, the diffusion coefficients for both fuel and oxidizer are assumed to be equal, so that the mass diffusion coefficient is simply denoted by D^* . Then the Lewis number, denoted by L , is the ratio of D_T^* to D^* and also is constant.

In terms of the heat release per unit mass of the oxidizer consumed, denoted by Q^* , and the specific heat at constant pressure, denoted by c_p^* , the nondimensional temperature T is defined as

$$T \equiv \frac{\sigma L c_p^* (T^* - T_{-\infty}^*)}{Q^* Y_{O-\infty}^*} = \frac{1}{q} \left(\frac{T^*}{T_{-\infty}^*} - 1 \right), \quad (1)$$

where T^* is the unscaled temperature, $Y_{O-\infty}^*$ the oxidizer mass fraction at the oxidizer boundary, $T_{-\infty}^*$ the temperature at that boundary, σ the stoichiometric mass ratio of oxidizer to fuel, and q the nondimensional heat-release parameter defined as $q \equiv Q^* Y_{O-\infty}^* / \sigma L c_p^* T_{-\infty}^*$. The subscripts ∞ and $-\infty$ refer to the fuel and oxidizer boundaries, respectively. The scaled fuel mass fraction Y_F and scaled oxidizer mass fraction Y_O are also defined as

$$Y_F = \sigma Y_F^* / Y_{O-\infty}^*, \quad Y_O = Y_O^* / Y_{O-\infty}^*. \quad (2)$$

Near the line of symmetry of axisymmetric counterflows, the scalar variables, such as Y_F , Y_O and T , are independent of the transverse coordinate, so that only the normal coordinate needs to be considered. In terms of D_T^* and the rate of strain a^* , the nondimensional normal distance from the stagnation plane is defined as

$$y \equiv y^* / (D_T^* / a^*)^{1/2}, \quad (3)$$

where $(D_T^* / a^*)^{1/2}$ is the characteristic thickness of the mixing layer. For chemistry, we consider an overall irreversible one-step Arrhenius reaction with reaction orders of unity for both fuel and oxidizer. Then the rate of fuel consumption per unit mass is given by

$$w^* = B^* Y_O^* Y_F^* \exp(-E^* / R^* T^*) \quad (4)$$

where B^* is a pre-exponential factor with units of reciprocal time, E^* the overall activation energy, and R^* the universal gas constant.

Under these assumptions, the conservation equations for species and energy can be written as

$$\frac{d^2 Y_F}{dy^2} + 2Ly \frac{dY_F}{dy} = Da Y_O Y_F \exp\left(-\frac{E^*}{R^* T_{-\infty}^*} \frac{1}{1 + qT}\right), \quad (5)$$

$$\frac{d^2 Z}{dy^2} + 2Ly \frac{dZ}{dy} = 0, \quad (6)$$

$$\frac{d^2 H}{dy^2} + 2y \frac{dH}{dy} = (1 - L)y \left[\frac{dY_F}{dy} + \frac{dY_O}{dy} \right], \quad (7)$$

where Da is the Damköhler number defined as $Da \equiv B^* L Y_{O-\infty}^* / a^*$. In Equations (6) and (7), the modified mixture fraction Z and the modified total enthalpy H are [9]

$$Z \equiv (Y_F - Y_O + 1)/(1 + A_F), \quad H \equiv T + (Y_O + Y_F)/2, \quad (8)$$

where an equivalence ratio A_F is given by $A_F \equiv \sigma Y_{F\infty}^* / Y_{O\infty}^*$. It is worthy of note that Z is a conserved scalar, because both Lewis numbers for fuel and oxidizer are equal. If two Lewis numbers are unequal, the equation for Z will have an inhomogeneous term, which will cause the distribution of Z to vary with leakage. With a known distribution of Y_F , Z and H describe the corresponding distributions of Y_O and T , respectively. Use of the coupling functions Z and H , instead of Y_O and T , enables us to determine the reaction-sheet location and the flame temperature without using jump conditions.

The applicable boundary conditions for Equations (5)–(7) are

$$\left. \begin{array}{lll} Y_F \rightarrow A_F, & Z \rightarrow 1, & H \rightarrow H_\infty = T_\infty + A_F/2 \quad \text{as } y \rightarrow \infty \\ Y_F \rightarrow 0, & Z \rightarrow 0, & H \rightarrow 1/2 \quad \text{as } y \rightarrow -\infty \end{array} \right\}. \quad (9)$$

Since the Arrhenius reaction is controlled mainly by the temperature distribution, it does not distinguish chemical differences between the fuel and oxidizer. Systems with $A_F \geq 1$ are symmetric to systems with $A_F \leq 1$ in the sense that they can be converted into each other by exchanging the fuel and oxidizer. Here we therefore consider only cases with $A_F \geq 1$.

3. Burke-Schumann solution

The first approximation to the diffusion-flame structure is the Burke-Schumann solution, which arises from infinite-rate chemistry, *i.e.* $Da \rightarrow \infty$. Under this limit, Equation (5) requires that $Y_O Y_F = 0$, that is, the fuel and oxidizer streams are separated by a reaction sheet.

As a first solution to the conservation equations, the modified mixture fraction is found to be

$$\bar{Z} = 1 - \frac{1}{2} \operatorname{erfc}(\sqrt{L}y), \quad (10)$$

where ‘erfc’ is the complementary error function [13] and the ‘bar’ denotes the Burke-Schumann solution. Since $\bar{Y}_F = \bar{Y}_O = 0$ at the reaction sheet, $\bar{Z}_s = 1/(1 + A_F)$ and the reaction-sheet location y_s is found to be

$$y_s = L^{-\frac{1}{2}} \operatorname{erfc}^{-1}(2A_F/(1 + A_F)), \quad (11)$$

where ‘erfc⁻¹’ is the inverse complementary error function, and the subscript s denotes quantities evaluated at the reaction sheet. The distribution of \bar{Y}_F is obtained from (5), and the corresponding distribution of \bar{Y}_O is subsequently obtained by substituting \bar{Y}_F and \bar{Z} in the definition of Z in (8). The profiles are then given by

$$\begin{aligned}\bar{Y}_F &= [A_F - \frac{1}{2}(1 + A_F) \operatorname{erfc}(\sqrt{L}y)]u(y - y_s), \\ \bar{Y}_O &= [\frac{1}{2}(1 + A_F) \operatorname{erfc}(\sqrt{L}y) - A_F]u(y_s - y),\end{aligned}\quad (12)$$

where $u(x)$ is the unit step function; $u(x) = 1$ for $x > 0$ and $u(x) = 0$ for $x < 0$.

Integrating (7), we find that the distribution of \bar{H} is given by

$$\bar{H} = \frac{1}{4} \operatorname{erfc}(y) + \left(\frac{A_F}{4} + \frac{T_\infty}{2} \right) [2 - \operatorname{erfc}(y)] - \int_{-\infty}^{\infty} h(\zeta) G(\zeta; y) d\zeta, \quad (13)$$

where the Green’s function $G(\zeta; y)$ is

$$G(\zeta; y) = \begin{cases} \frac{1}{4} \sqrt{\pi} \exp(\zeta^2) [2 - \operatorname{erfc}(\zeta)] \operatorname{erfc}(y) & \text{for } y > \zeta \\ \frac{1}{4} \sqrt{\pi} \exp(\zeta^2) \operatorname{erfc}(\zeta) [2 - \operatorname{erfc}(y)] & \text{for } y < \zeta \end{cases} \quad (14)$$

and $h(y)$ is the inhomogeneous term of Equation (7), given by

$$\begin{aligned}h(y) &= (1 - L)y \left[\frac{dY_F}{dy} + \frac{dY_O}{dy} \right] \\ &= (1 + A_F)(1 - L) \sqrt{L/\pi} y \exp(-Ly^2) \operatorname{sgn}(y - y_s).\end{aligned}\quad (15)$$

As a part of the solution for $\bar{H}(y)$, \bar{H}_s is of particular interest. Since $\bar{Y}_F = \bar{Y}_O = 0$ at $y = y_s$, \bar{H}_s corresponds to the nondimensional Burke-Schumann flame temperature and is given by

$$\bar{H}_s = \frac{1}{4}(1 + A_F)L^{1/2} \exp[(1 - L)y_s^2] \operatorname{erfc}(y_s) [2 - \operatorname{erfc}(y_s)] + \frac{1}{2}T_\infty [2 - \operatorname{erfc}(y_s)]. \quad (16)$$

In addition, the gradients of \bar{H} and \bar{Z} at the reaction sheet are

$$\begin{aligned}\bar{H}'_s &= -\frac{1}{2}(1 + A_F)(L/\pi)^{1/2} \exp(-Ly_s^2) [1 - \operatorname{erfc}(y_s)] + T_\infty \pi^{-1/2} \exp(-y_s^2), \\ \bar{Z}'_s &= \sqrt{L/\pi} \exp(-Ly_s^2),\end{aligned}\quad (17)$$

which will be used to scale the inner-layer analysis.

4. AEA analysis

Once the rate of the chemical reaction becomes finite, *i.e.*, the Damköhler number Da becomes finite, the reactants begin to leak through the reaction sheet. Taking the Zel’dovich number β , defined as

$$\beta \equiv \frac{E^*}{R^*T_\infty} \frac{q\bar{H}_s}{(1 + q\bar{H}_s)^2}, \quad (18)$$

to be the large expansion parameter, we are concerned with a limit in which the magnitude of the reactant leakage is of order β^{-1} . This distinguished limit defines the diffusion-flame regime or near-equilibrium regime [2]. Here we pose the problem of finding the value of Da corresponding to a specified value of fuel or oxidizer leakage of order β^{-1} .

4.1. OUTER CONVECTIVE-DIFFUSIVE LAYER ANALYSIS

In this layer, the scalar variables are perturbed from their Burke-Schumann distributions in the form

$$\left. \begin{aligned} Y_i^{\text{out}} &= \bar{Y}_i + \beta^{-1} \bar{H}_s \hat{Y}_i + \dots \\ Z^{\text{out}} &= \bar{Z} + \beta^{-1} \bar{H}_s \hat{Z} + \dots \\ H^{\text{out}} &= \bar{H} + \beta^{-1} \bar{H}_s \hat{H} + \dots \end{aligned} \right\} \text{for } i = O, F. \quad (19)$$

If Y_F , Z and H are replaced by \hat{Y}_F , \hat{Z} and \hat{H} , the conservation equations for \hat{Y}_F , \hat{Z} and \hat{H} are identical to (5)–(7) with a vanishing reaction term. The applicable boundary conditions are homogenous at both boundaries.

The solution for \hat{Z} is found to be identically zero throughout the flow, so that \hat{Y}_F is equal to \hat{Y}_O . Then, \hat{Y}_F and \hat{Y}_O may be written as

$$\hat{Y}_F = \hat{Y}_O = \begin{cases} \hat{Y}^{+\frac{1}{2}}(1 + A_F)A_F^{-1} \operatorname{erfc}(\sqrt{L}y) & \text{for } y > y_s \\ \hat{Y}^{-\frac{1}{2}}(1 + A_F)[2 - \operatorname{erfc}(\sqrt{L}y)] & \text{for } y < y_s \end{cases}, \quad (20)$$

where $\hat{Y}^\pm = \hat{Y}_F(y_s^\pm) = \hat{Y}_O(y_s^\pm)$. Here \hat{Y}^+ represents the oxidizer leakage, while \hat{Y}^- does the fuel leakage. The leakages \hat{Y}^\pm are still unknown and will be determined later by matching with the inner-layer solution.

In a manner similar to how \bar{H}_s was obtained, \hat{H}_s is found in terms of \hat{Y}^\pm to be

$$\hat{H}_s = C_s^- \hat{Y}^- + C_s^+ \hat{Y}^+ \quad (21)$$

where the constant C_s^\pm are

$$\begin{aligned} C_s^- &= \left\{ \frac{1}{2} + [(1 - L)y_s \frac{1}{2} \sqrt{\pi} - (1 + A_F) \frac{1}{4} \sqrt{L} \exp(-Ly_s^2)] \right. \\ &\quad \left. \exp(y_s^2) [2 - \operatorname{erfc}(y_s)] \right\} \operatorname{erfc}(y_s), \\ C_s^+ &= \left\{ \frac{1}{2} - [(1 - L)y_s \frac{1}{2} \sqrt{\pi} + \frac{1 + A_F}{A_F} \frac{1}{4} \sqrt{L} \exp(-Ly_s^2)] \right. \\ &\quad \left. \exp(y_s^2) \operatorname{erfc}(y_s) \right\} [2 - \operatorname{erfc}(y_s)]. \end{aligned} \quad (22)$$

Here \hat{H}_s may be called the excess enthalpy, since it measures an excess or deficiency of the total enthalpy at the reaction sheet from \bar{H}_s . As the reactants leak through the reaction sheet, the chemical enthalpy in the reaction zone increases from that of the Burke-Schumann limit, while the thermal enthalpy decreases. Depending on whether the Lewis number L is less than or greater than unity, an excess or deficiency in the total enthalpy may occur. If the Lewis

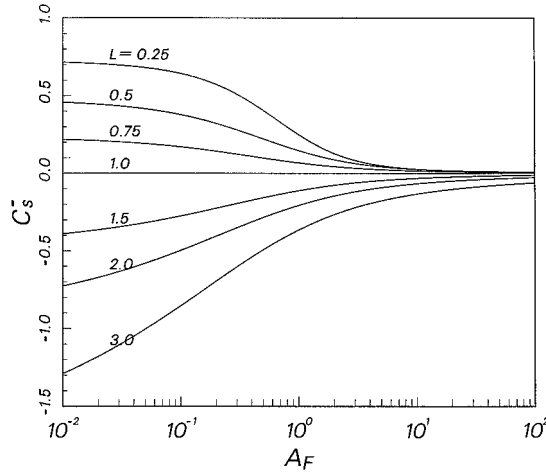


Figure 2. Variation of C_s^- with A_F for various values of L .

number is unity, the increase of the chemical energy is exactly balanced by a decrease of the thermal energy, so that $\hat{H}_s = 0$. However, if the Lewis number is less than unity, then the loss of the thermal enthalpy is smaller than the gain of the chemical energy, because of a weaker heat loss toward the colder boundaries, contributed by the effect of smaller thermal diffusivity compared with mass diffusivity. Therefore, the excess enthalpy \hat{H}_s becomes positive for $L < 1$, while \hat{H}_s is negative for $L > 1$. It is the excess enthalpy \hat{H}_s that causes the extinction characteristics of flames with nonunity Lewis numbers to become different from those of flames with unity Lewis number.

Figure 2 shows variation of C_s^- with A_F and L obtained from (11) and (22). The corresponding value of C_s^+ can also be found from Figure 2 because $C_s^+(A_F) = C_s^-(1/A_F)$. If the ratio A_F is unity, then $C_s^\pm = (1 - \sqrt{L})/2$. In addition, for $A_F \rightarrow \infty$, the asymptotic behaviors of C_s^\pm are found to be $C_s^- \rightarrow 0$ and $C_s^+ \rightarrow 1 - L$. On the other hand, $C_s^- \rightarrow 1 - L$ and $C_s^+ \rightarrow 0$ as $A_F \rightarrow 0$.

In real combustion systems, Lewis numbers for fuel and oxidizer are seldom equal or constant, so that evaluation of the excess enthalpy \hat{H}_s , *i.e.* evaluation of the constants C_s^\pm , becomes algebraically more complicated and tedious. Then it is useful to calculate numerically the values of C_s^\pm . To do so, we first set $\hat{Y}_F^- = 1$ and $\hat{Y}_F^+ = 0$, and then calculate the profile of \hat{Y}_F . The differential equation for \hat{Y}_F is identical to (5) with the homogeneous right-hand side, $L = L_F$ and Y_F being replaced by \hat{Y}_F . The profile of \hat{Y}_O can be found from the subsequent numerical integration of \hat{Z} and the coupling relation in Equation (8). However, it must be noted that, if $L_F \neq L_O$, the differential equation for \hat{Z} is no longer homogeneous, *i.e.* there exists an additional transient-convective term of \hat{Y}_F in the right-hand side of (6), so that \hat{Z} is no longer identically zero. Finally, numerical integration for \hat{H} , the differential equation of which is given in (7) with ‘hat’ placed on all state variables and \hat{Y}_F and \hat{Y}_O separately taking $L = L_F$ and $L = L_O$, yields the profile of \hat{H} . Then the value of \hat{H}_s corresponds to the value of C_s^- . In a manner similar to how we obtained C_s^- , we can find the value of C_s^+ by setting $\hat{Y}_F^- = 0$ and $\hat{Y}_F^+ = 1$. Similarly, \hat{Z}_s can be expressed as a linear combination of \hat{Y}_F^\pm .

4.2. INNER REACTIVE-DIFFUSIVE LAYER ANALYSIS

In the reactive-diffusive layer, the scalar variables are perturbed from their values of the Burke-Schumann solution at the reaction sheet. Since $\bar{Y}_F = 0$ at $y = y_s$, the inner expansion for Y_F is given by

$$Y_F^{\text{in}} = \beta^{-1} \bar{H}_s(\phi + \xi), \quad (23)$$

where the stretched inner coordinate ξ is given by

$$\xi = A\beta(y - y_s) - \lambda. \quad (24)$$

The constant factors A and λ will be adjusted later to simplify the inner-layer analysis. The previous solution for the modified mixture fraction Z is valid, even in the reaction zone, because Z is a coupling function. Then, in terms of the inner variable ξ , Z is expanded near $y = y_s$ as

$$Z = (1 + A_F)^{-1} + \beta^{-1}[\bar{Z}'_s \xi/A + \bar{H}_s \hat{Z}_s + \bar{Z}'_s \lambda/A] + \dots \quad (25)$$

Substituting (23) and (25) in the first of Equation (8), we find that the inner expansion of Y_O is given by

$$Y_O^{\text{in}} = \beta^{-1} \bar{H}_s[\phi + (1 - A^{-1}(1 + A_F)H_s^{-1} \bar{Z}'_s)\xi - (1 + A_F)(\hat{Z}_s + \bar{Z}'_s A^{-1} H_s^{-1} \lambda)] + \dots \quad (26)$$

Here we can adjust the two free parameters A and λ to obtain a simpler functional form for Y_O^{in} . Upon choosing

$$Y_O^{\text{in}} = \beta^{-1} \bar{H}_s(\phi - \xi) \quad (27)$$

we find the two parameters to be

$$A = \frac{1}{2}(1 + A_F) \bar{Z}'_s \bar{H}_s^{-1}, \quad \lambda = -\frac{1}{2}(1 + A_F) \hat{Z}_s = 0. \quad (28)$$

For this analysis, translation of the inner variable from y_s by an amount λ is not necessary because $\hat{Z}_s = 0$. However, if $\hat{Z}_s \neq 0$, which occurs for $L_O \neq L_F$, such a translation is essential if we are to fix the coordinate where $Y_F = Y_O$. If this is not done, then the resulting inner-layer equation may explicitly include the leakage terms, so that the solution of the inner-layer equation becomes more complicated.

The inner expansion of the temperature is obtained in a manner similar to that employed in obtaining (27). Expanding H in terms of ξ and substituting the resulting expansion and (23) and (27) in the second of Equation (8), we find

$$T^{\text{in}} = \bar{H}_s - \beta^{-1} \bar{H}_s(\phi + \gamma\xi - \hat{H}_s), \quad (29)$$

where γ is a heat-loss parameter, defined as

$$\gamma \equiv -\frac{2\bar{H}'_s}{(1 + A_F)\bar{Z}'_s} = 1 - \text{erfc}(y_s) - \frac{2T_\infty}{(1 + A_F)\sqrt{L}} \exp[(L - 1)y_s^2]. \quad (30)$$

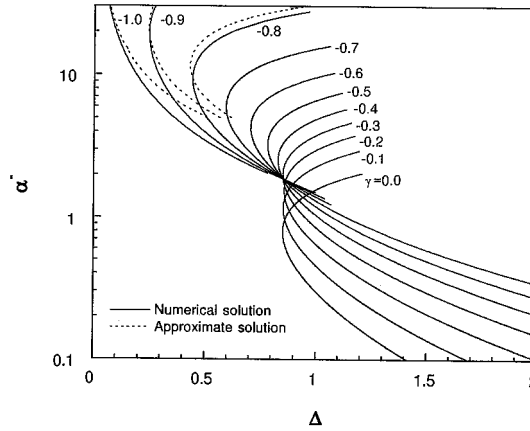


Figure 3. Variation of Δ with α^- for various values of γ .

If $\gamma = 0$, then the rates of heat losses are equal at both boundaries, while $\gamma = 1(-1)$ corresponds to adiabatic conditions at the oxidizer (fuel) boundary.

Substituting (23), (27) and (29) in Equation (5), we obtain the governing equation for the inner layer as

$$\frac{d^2\phi}{d\xi^2} = \Delta(\phi + \xi)(\phi - \xi) \exp[-(\phi + \gamma\xi)], \tag{31}$$

$$\frac{d\phi}{d\xi} \rightarrow \pm 1 \quad \text{as} \quad \xi \rightarrow \pm\infty,$$

where the reduced Damköhler number Δ is defined as

$$\Delta \equiv Da \exp(\hat{H}_s) \frac{4\bar{H}_s^3}{(1 + A_F)^2 \bar{Z}_s'^2 \beta^3} \exp\left(-\frac{E^*}{R^* T_{-\infty}^*} \frac{1}{1 + q\bar{H}_s}\right). \tag{32}$$

We obtain the boundary conditions in (31) by matching with the slopes of the Burke-Schumann solution. The simple boundary conditions in (31) are obtained as a result of an adjustment of the parameters A and λ . The above inner problem was initially derived and solved by Liñán [2]. For a suitably specified value of Δ , there exists a unique set of the leakage parameters, defined by

$$\alpha^- = (\phi + \xi)_{-\infty}, \quad \alpha^+ = (\phi - \xi)_{\infty}. \tag{33}$$

Figure 3 shows some of the numerical results for Δ as a function of α^- . It is seen from Figure 3 that there is a minimum value of Δ , below which no solution exists. Although Figure 3 is for negative γ , we may apply the figure also for $\gamma > 0$ by replacing α^- with α^+ , because the inner problem is symmetric in γ . In addition, the derivative of Δ with respect to α^- , which is required to determine the extinction condition, is shown in Figure 4. Appendix A provides further details on how the curves in Figure 4 are computed.

To complete the analysis, the leakage parameters α^- and α^+ are matched to the first-order outer solution to yield

$$\hat{Y}^- = \alpha^-, \quad \hat{Y}^+ = \alpha^+, \tag{34}$$

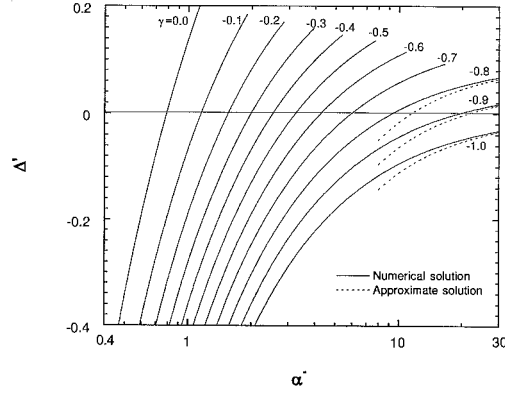


Figure 4. Variation of Δ' with α^- for various values of γ .

which shows that α^- and α^+ are proportional to the fuel and oxidizer leakages, respectively. Then, the Damköhler number Da is given as

$$Da = \Delta \exp(-\hat{H}_s) \frac{(1 + A_F)^2 \bar{Z}_s'^2 \beta^3}{4\bar{H}_s^3} \exp\left(\frac{E^*}{R^* T_{-\infty}^*} \frac{1}{1 + q\bar{H}_s}\right), \quad (35)$$

where the excess enthalpy \hat{H}_s is

$$\hat{H}_s = C_s^- \alpha^- + C_s^+ \alpha^+. \quad (36)$$

From (35) and (36), the Damköhler number Da can then be expressed as a function of the fuel-leakage parameter α^- .

If the equivalence ratio A_F is sufficiently large, which is typical of many diffusion flames, then the value of γ approaches negative unity, as is seen from Equation (30). Under this circumstance, the larger leakage parameter corresponds to α^- , because the reaction becomes rapidly frozen in the oxidizer stream in which the heat loss is greater. Moreover, the oxidizer leakage to the fuel boundary vanishes, so that the dominant contribution to the excess enthalpy is associated with the fuel leakage.

5. Quasisteady extinction condition

To calculate the Damköhler number, first we solve for the Burke-Schumann flame structure with the boundary conditions and physical parameters. Then, all the terms in (35), except for Δ and \hat{H}_s , are determined from the Burke-Schumann solution. For γ already determined by (30), a value of the fuel-leakage parameter α^- is chosen to find Δ and α^+ from the numerical solution of (31). The Damköhler number corresponding to the specified value of α^- is found from (35) and (36). Repeating the same procedure for different values of α^- , we find the variation of the Damköhler number Da as a function of α^- . It is essential to guard against overlooking the factor $\exp(\hat{H}_s)$ in Equation (35). Ignoring this excess enthalpy contribution has led to errors in certain earlier works.

As α^- initially increases from zero, Da decreases from infinity, and Da may exhibit a minimum at a certain value of α^- . Any further increase of α^- from the minimum condition leads to an increase of Da . If a value of Da is less than the minimum value of Da , denoted by

Da_E , there exists no steady solution to our problem. Therefore, the minimum condition can be identified as the quasisteady extinction condition, and Da_E is referred to as the extinction Damköhler number.

The simplest extinction analysis is encountered when $L = 1$. Since the excess enthalpy \hat{H}_s is zero for $L = 1$, the only term in (35) that is dependent on the fuel-leakage parameter α^- is the reduced Damköhler number Δ . Therefore, the quasisteady extinction condition corresponds to the minimum of Δ . An approximation for the minimum Δ , denoted by Δ_m , as a function of the heat-loss parameter γ is given by Liñán [2] as

$$\Delta_m = e[(1 - |\gamma|) - (1 - |\gamma|)^2 + 0.26(1 - |\gamma|)^3 + 0.055(1 - |\gamma|)^4]. \quad (37)$$

Once the Burke-Schumann flame structure is determined, we can easily obtain the extinction Damköhler number from (35) and (37), even without performing the AEA analysis presented in the previous section.

However, if $\hat{H}_s \neq 0$, *i.e.* $L \neq 1$, then the Δ_m correlation in Equation (37) is no longer valid for the quasisteady extinction condition, and variation of the excess enthalpy \hat{H}_s with α^- must be taken into account to calculate the extinction Damköhler number. It is the variation of \hat{H}_s , which was not properly included in some previous analyses for diffusion-flame extinction with nonunity Lewis number. Differentiating the logarithm of (35) with respect to α^- , we find that

$$\frac{1}{Da} \frac{dDa}{d\alpha^-} = \Delta' - C_s^- - rC_s^+, \quad (38)$$

where

$$\Delta' \equiv \frac{1}{\Delta} \frac{d\Delta}{d\alpha^-}, \quad r \equiv \frac{d\alpha^+}{d\alpha^-}. \quad (39)$$

Then the minimum of Da leads to the quasisteady extinction condition in the form

$$\Delta' = C_s^- + rC_s^+. \quad (40)$$

Here it should be kept in mind that the quasisteady extinction condition corresponds to the minimum of the Damköhler number, not to the minimum of the reduced Damköhler number given in (37), although other conditions, such as onset of instability, sometimes are related to the minimum of the reduced Damköhler number [12].

For $L < 1$, the right-hand side of (40) is always positive, because C_s^\pm and r are positive definite. Then, the quasisteady extinction occurs with $\Delta' > 0$, that is, beyond the turning point of the $\Delta - \alpha^-$ curve. As α^- increases from its value at the turning point, denoted by α_m^- , the reduced Damköhler number Δ begins to increase. However, Da continues to decrease, because the additional gain of the excess enthalpy strengthens the chemical reaction, so that the flame becomes resistant to extinction. The quasisteady extinction can occur only when the increase of Δ with increasing α^- is large enough to overcome the effect of the excess enthalpy. As a consequence, diffusion flames with $L < 1$ extinguish with fuel leakage greater than α_m^- . On the other hand, for $L > 1$, extinction occurs before the $\Delta - \alpha^-$ turning point.

To illustrate the effects of \hat{H}_s on the quasisteady extinction, sample calculations are reported here. In two previous flame-structure analyses with nonunity Lewis numbers [7, 8], the excess-enthalpy effects were not taken into account, while the Burke-Schumann solutions

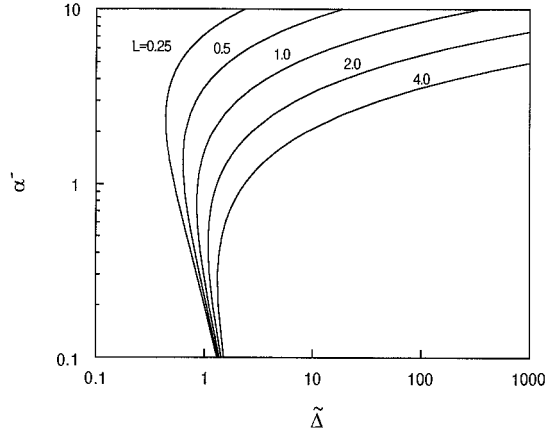


Figure 5. Variation of $\tilde{\Delta}$ with α^- for $A_F = 1, T_\infty = 0$ and $L = 0.25, 0.5, 1, 2, 4$.

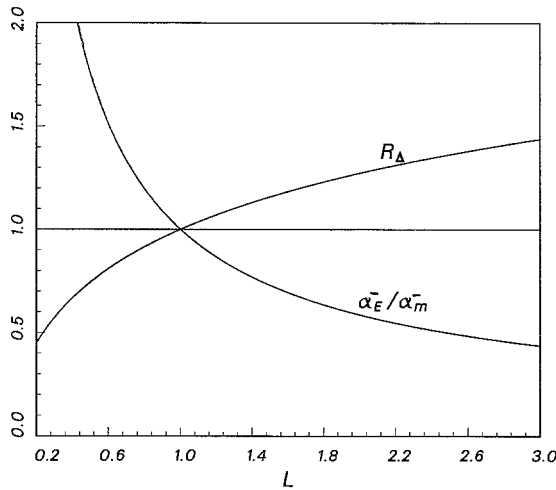


Figure 6. Variation of the ratio $R_\Delta = \tilde{\Delta}_E/\Delta_m$ and α_E^-/α_m^- with L for the case of $A_F = 1$ and $T_\infty = 0$.

were properly obtained. Since the effects of the reactant leakage appear only in the reduced Damköhler number Δ and the excess enthalpy \hat{H}_s , it is more convenient to introduce a modified reduced Damköhler number $\tilde{\Delta}$ as

$$\tilde{\Delta} = \Delta \exp(-\hat{H}_s). \tag{41}$$

Then, quasisteady extinction occurs at the minimum of $\tilde{\Delta}$, denoted by $\tilde{\Delta}_E$, and the inaccuracy introduced by neglecting the effects of the excess enthalpy arises only from the difference of the factor $\tilde{\Delta}_E$ from Δ_m .

An example is demonstrated for $A_F = 1$ and $T_\infty = 0$. We find that $y_s = 0, C_s^\pm = (1 - \sqrt{L})/2, \gamma = 0$, and $r = 1$. The quasisteady extinction condition becomes $\Delta' = 1 - \sqrt{L}$. Figure 5 shows the variation of $\tilde{\Delta}$ with the leakage parameter α^- for various values of the Lewis number. For the case of $L = 1$, the turning point of $\tilde{\Delta}$ occurs at $\tilde{\Delta}_E = \Delta_m$. As the Lewis number decreases, turning of $\tilde{\Delta} - \alpha^-$ curve is delayed to a smaller value of $\tilde{\Delta}_E$, *i.e.* a larger value of strain rate. For a range of the Lewis number, Figure 6 shows the ratio of

the value of $\tilde{\Delta}_E$ to Δ_m , denoted by $R_\Delta = \tilde{\Delta}_E/\Delta_m$, which represents the inaccuracy of the extinction Damköhler number when the excess enthalpy is neglected. It is seen from Figure 6 that the extinction Damköhler number can be in error by a factor of two. Also seen in Figure 6 is the ratio of the extinction fuel-leakage parameter α_E^- to α_m^- , showing that α_E^- is much larger than α_m^- for $L < 1$.

In order to use Equation (40), the quasisteady extinction condition has to be found by an iterative method from numerical results for the functions $\Delta(\alpha^-)$ and $\alpha^+(\alpha^-)$. However, if the heat-loss parameter γ is close to -1 or 1 , the relationship for $\Delta(\alpha^-)$ can be approximated in an explicit functional form [14], so that the quasisteady extinction condition can be readily calculated. For most practical diffusion-flame configurations, the equivalence ratio A_F is extremely large (usually greater than 10), so that $\gamma \rightarrow -1$ turns out to be a realistic approximation. Under this limit, the oxidizer leakage is negligible, while the fuel-leakage parameter α^- near extinction is much greater than unity. From a distinguished limit in which $(\gamma + 1) \ll 1$ and $\alpha^- \gg 1$, Appendix B derives the asymptotic relation for $\Delta(\alpha^-)$ as

$$\Delta = 2(\alpha^-)^{-1} \exp(m\alpha^-)[1 - 3.3440m + 1.3440(\alpha^-)^{-1}], \quad (42)$$

where m is a new heat-loss parameter defined as $m \equiv (1 - |\gamma|)/2$ and $m > 0$ ($m < 0$) corresponds to subadiabaticity (superadiabaticity) of the flame to the fuel stream. From this result, the quasisteady extinction condition in (40) can be rewritten as

$$\alpha_E^- = \frac{1 + 1.3440(m - C_s^-)}{m - C_s^-}, \quad (43)$$

which gives the two-term approximation to α_E^- . Substituting this approximation in (41) and (42), we find the two-term approximation to $\tilde{\Delta}$ at quasisteady extinction as

$$\tilde{\Delta}_E = 2e(m - C_s^-)(1 - 2m - 1.3440C_s^-) \quad (44)$$

and the extinction Damköhler number is given by

$$Da_E = \tilde{\Delta}_E \frac{(1 + A_F)^2 \bar{Z}_s'^2 \beta^3}{4\bar{H}_s^3} \exp\left(\frac{E^*}{R^*T_{-\infty}^*} \frac{1}{1 + q\bar{H}_s}\right). \quad (45)$$

Equations (42) through (44) become applicable for $\gamma \rightarrow 1$ when we replace α^- and C_s^- with α^+ and C_s^+ . If a flame is nearly adiabatic, we can obtain the quasisteady extinction condition from (44) without performing the inner-layer analysis.

The limit of $m \rightarrow 0$ usually occurs when the ratio A_F is a large value. As can be seen from Figure 2, C_s^- also approaches zero. Under this circumstance, the leading-order approximation to $\tilde{\Delta}_E$ becomes $\tilde{\Delta}_E = 2e(m - C_s^-)$. Since $\Delta_m = 2em$, the ratio of the extinction Damköhler numbers R_Δ , defined in the previous paragraph for Figure 6, becomes $R_\Delta = (m - C_s^-)/m$. Therefore, inaccuracy of the extinction Damköhler number depends on how close the factor C_s^- is to m .

In (43) and (44), it should be noted that only the positive solution for α_E^- or $\tilde{\Delta}_E$ is meaningful. Otherwise, there is no quasisteady extinction condition until the fuel-leakage parameter α^- is increased from zero to infinity. The necessary condition that $\tilde{\Delta}_E$ is positive corresponds to $m > C_s^-$. Considering the case of $L = 1$, i.e. $C_s^- = 0$, we observe that this

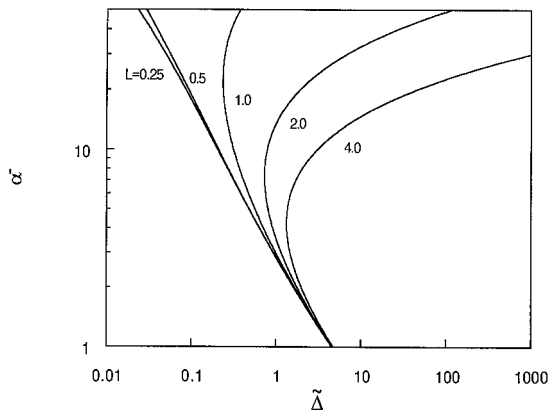


Figure 7. Variation of $\tilde{\Delta}$ with α^- for $A_F = 20$, $T_\infty = 0$ and $L = 0.25, 0.5, 1, 2, 4$.

necessary condition says that extinctions are achieved only for subadiabatic flames ($m > 0$), which confirms the conventional result [2] that $Da_E \rightarrow 0$ as $m \rightarrow 0$. However, for $L < 1$, there exists a small range of the positive heat-loss parameter m that does not give any positive solution for α_E^- . Consequently, for $L < 1$, quasisteady extinctions cannot be found, unless the flame is sufficiently subadiabatic at both boundaries. On the other hand, for $L > 1$, superadiabatic flames with the parameter m in the range of $C_s^- < m < 0$ can be extinguished.

Variation of $\tilde{\Delta}$ with α^- for $L = 0.25, 0.5, 1, 2, 4$ is shown in Figure 7 for the case of $T_\infty = 0$ and $A_F = 20$, which is a typical equivalence ratio for hydrocarbon-air flames. Contrary to the flames in Figures 5 and 6, in which $A_F = 1$ and heat losses are equal at both boundaries, the flames in this case are slightly subadiabatic. It is apparent from Figure 7 that the effect of the excess enthalpy is even greater for nearly adiabatic flames. As comparison of $\tilde{\Delta}_E$ with Δ_m shows, for the case of $L = 4$, exclusion of the excess-enthalpy effect gives rise to the inaccuracy of the extinction Damköhler number by a factor of five for $A_F = 20$, while the inaccuracy for $A_F = 1$ is not greater than a factor of two. In addition, there are no turning points for $L = 0.25$ and 0.5 . The numerical calculations for these Lewis numbers show that the values of C_s^- are greater than the values of m , so that the excess enthalpy effect is strong enough to overcome the heat losses. Consequently, extinctions do not occur for these flames. However, it must be kept in mind that the above results, namely that subadiabatic flames with Lewis numbers less than unity may not be extinguished, need to be carefully interpreted. Since many assumptions, such as equal Lewis numbers and one-step Arrhenius reaction rate, are introduced in the model flames, the actual subadiabatic range, in which realistic flames are not extinguished, will be quite different from that in the model flames in Figure 7.

6. Concluding remarks

In this work, we analyzed quasisteady extinction of diffusion flames with nonunity Lewis numbers by employing a counterflow diffusion flame as a model. Of particular interest was an investigation of the effects of excess enthalpy on extinction characteristics of diffusion flames. From the analysis employing the diffusion-flame regime of activation-energy asymptotics, the results showed that diffusion flames with Lewis number less than unity are more robust, because a gain of the excess enthalpy by the weaker diffusive loss of the thermal energy strengthens the chemical reaction. The resulting extinction Damköhler numbers can differ by

an amount of order unity from values obtained without consideration of the effects of the excess enthalpy.

Influences of the excess enthalpy will also appear in extinction analyses with full or reduced chemical kinetics. Since the asymptotic analyses with reduced chemistry are developed to obtain quantitatively reasonable extinction conditions, a neglect of the effects of excess enthalpy may lead to inaccuracies in the asymptotic results. In particular, hydrogen-air flames near extinction are expected to be most significantly influenced by excess enthalpy because, besides leakage of hydrogen, leakage of the H radical can be significant throughout the flow [15]. Unlike hydrogen-air flames, hydrocarbon-air flames may not be affected by excess enthalpy. The results of rate-ratio asymptotics for methane-air flames [16] show that, although the heat loss to the fuel stream is smaller, oxygen leaks through the reaction zone, because of the faster fuel-consumption step compared with the oxidization step. Because the Lewis number of oxygen is close to unity, the excess enthalpy is not likely to be large enough to modify the reaction rate significantly for these flames.

Errors in the extinction Damköhler number by a factor of two would be acceptable in some applications. However, the shift of the quasisteady extinction condition from the minimum condition of Δ is found to be important for dynamics of near-extinction flames. A recent analysis concerning diffusional-thermal instability of diffusion flames [12] has shown that instability can be initiated between the minimum condition of Δ and the quasisteady extinction condition for $L < 1$. In addition, for $L > 1$, planar disturbances are suspected to become unstable even before quasisteady extinction [10, 12], so that the real extinction condition might correspond to a point at which the planar instability is initiated. Because of these complicating possibilities, the extinction condition identified by the minimum of the Damköhler number is strictly termed quasisteady condition in the present analysis. Further studies are necessary to understand fully the dynamic behaviors of near-extinction diffusion flames.

Appendix 1. Method for calculating the derivative of the inner-layer structure with respect to reactant leakage

Results for Δ' are needed in Equation (40) to identify the quasisteady extinction condition. In principle, we may obtain Δ' by solving the inner problem in (31) for various values of α^- and then differentiating numerically. However, this is quite inaccurate, and therefore the alternative procedure described here is employed.

In order to calculate various derivatives of the inner-layer flame structure with respect to α^- , Equation (31) is expanded to include a small perturbation of α^- about a given value α_0^- . Since ϕ and Δ are parametrically dependent on α^- , letting $\tilde{\alpha}$ a small increment in α^- , we may write

$$\left. \begin{aligned} \alpha^- &= \alpha_0^- + \tilde{\alpha} \\ \phi(\xi; \alpha^-) &= \phi(\xi; \alpha_0^-) + \frac{\partial \phi}{\partial \alpha^-}(\xi; \alpha_0^-) \tilde{\alpha} + \cdots = \phi(\xi; \alpha_0^-) + \Phi(\xi; \alpha_0^-) \tilde{\alpha} + \cdots \\ \Delta(\alpha^-) &= \Delta(\alpha_0^-) + \left. \frac{d\Delta}{d\alpha^-} \right|_{\alpha_0^-} \tilde{\alpha} + \cdots = \Delta_0(1 + \Delta' \tilde{\alpha} + \cdots) \end{aligned} \right\}, \quad (\text{A.1})$$

where

$$\Phi \equiv \left. \frac{\partial \phi}{\partial \alpha^-} \right|_{\alpha_0^-}, \quad \Delta' \equiv \left. \frac{d \log \Delta}{d \alpha^-} \right|_{\alpha_0^-}, \quad \Delta_0 \equiv \Delta(\alpha_0^-). \quad (\text{A.2})$$

Substituting the above expansions in (31) and collecting the terms at order $\tilde{\alpha}$ alone, we find the problem for determining Φ and Δ' to be

$$\left. \begin{aligned} \frac{d^2\Phi}{d\xi^2} &= \Delta_0 \exp[-(\phi + \gamma\xi)][(2\phi - \phi^2 + \xi^2)\Phi + \Delta'(\phi^2 - \xi^2)] \\ \frac{d\Phi}{d\xi} &\rightarrow 0 \quad \text{as } \xi \rightarrow \pm\infty \end{aligned} \right\}. \quad (\text{A.3})$$

To Equation (A.3) must be appended a supplementary condition to assure that the matching condition for the fuel leakage, $\alpha^- = (\phi + \xi)_{-\infty}$, is satisfied,

$$\alpha_0^- + \tilde{\alpha} = \phi(\xi; \alpha_0^-) + \Phi\tilde{\alpha} + \xi \quad \text{as } \xi \rightarrow -\infty. \quad (\text{A.4})$$

Since $\alpha_0^- = [\phi(\alpha_0^-) + \xi]_{-\infty}$, the applicable supplementary condition for (A.3) is then found to be

$$\Phi \rightarrow 1 \quad \text{as } \xi \rightarrow -\infty. \quad (\text{A.5})$$

Solution to (A.3) with the additional condition in (A.5) yields a unique function Φ , the eigenvalue Δ' and a constant value for $\Phi(\infty)$, which corresponds to $r = d\alpha^+/d\alpha^-$. For $\gamma = 0$, it is found that $r = 1$ for all values of α^- . The value of r decreases as γ decreases and approaches zero as γ approaches negative unity. Corresponding numerical results for Δ' are shown in Figure 4 as a function of α^- for various values of γ .

Appendix 2. Approximate inner solution for near-adiabatic case

If the ratio A_F is large, then the flame is nearly adiabatic on the fuel side, *i.e.* the heat-loss parameter γ is close to negative unity. In this limit, the inner solution can be approximated through an asymptotic approach. The procedure shown here follows a previous analysis by Clavin and Liñán [14].

To proceed with the analysis, it is convenient to introduce new variables as

$$\eta = 2\xi - \alpha^- = 2\xi - a/m, \quad \varphi = \phi - \xi, \quad (\text{A.6})$$

where an alternative heat-loss parameter m is defined as

$$m \equiv (1 - |\gamma|)/2 \quad (\text{A.7})$$

and the fuel-leakage parameter α^- is rescaled according to $\alpha^- = a/m$ with a being of order unity. For small values of m , we pose the problem of finding the reduced Damköhler number Δ that corresponds to a given value of the rescaled fuel-leakage parameter a . Upon substitution of the new variables of (A.6) in (31), the inner equation becomes

$$\begin{aligned} \frac{d^2\varphi}{d\eta^2} &= \Delta \frac{ae^{-a}}{4m} \varphi \exp[-(\varphi + m\eta)] \left[1 + \frac{m}{a}(\varphi + \eta) \right], \\ \varphi_\infty &\rightarrow 0, \quad (\varphi + \eta)_{-\infty} \rightarrow 0. \end{aligned} \quad (\text{A.8})$$

With the parameter m taken as the small expansion parameter, the solution is sought in the form

$$\left. \begin{aligned} \varphi &= \varphi_0 + m\varphi_1 + \dots \\ \delta &= \Delta \frac{ae^{-a}}{4m} = \delta_0(1 + md + \dots) \end{aligned} \right\}, \quad (\text{A.9})$$

where φ_0, φ_1 as well as δ_0 and d are assumed to be of order unity.

At the leading order, we find

$$\frac{d^2\varphi_0}{d\eta^2} = \delta_0\varphi_0 \exp(-\varphi_0), \quad (\text{A.10})$$

$$\varphi_0 \rightarrow 0 \text{ as } \eta \rightarrow \infty, \quad \varphi_0 + \eta \rightarrow 0 \text{ as } \eta \rightarrow -\infty.$$

The energy integral of the above equation results in $\delta_0 = 1/2$, thereby providing the first approximation to Δ ,

$$\Delta = 2ma^{-1} \exp(a) = 2 \exp(m\alpha^-)^{-1}/\alpha^- \quad (\text{A.11})$$

The equation for φ_1 of the expansion in (A.8) is

$$\frac{d^2\varphi_1}{d\eta^2} = \delta_0\varphi_0 \exp(-\varphi_0) \left[\varphi_1 \left(\frac{1}{\varphi_0} - 1 \right) + d - \eta + \frac{\varphi_0 + \eta}{a} \right], \quad (\text{A.12})$$

$$\varphi_1 \rightarrow 0 \text{ as } \eta \rightarrow \pm\infty.$$

Multiplying the both sides of this equation by $d\varphi_0/d\eta$, we find that

$$\frac{d}{d\eta} \left[\frac{d\varphi_0}{d\eta} \frac{d\varphi_1}{d\eta} - \frac{d^2\varphi_0}{d\eta^2} \varphi_1 \right] = \delta_0\varphi_0 \exp(-\varphi_0) \frac{d\varphi_0}{d\eta} \left[\varphi_1 \left(\frac{1}{\varphi_0} - 1 \right) + d - \eta + \frac{\varphi_0 + \eta}{a} \right], \quad (\text{A.13})$$

which can be integrated from $-\infty$ to ∞ to yield

$$-\frac{d}{2} - 1 + \nu \frac{1-a}{2a} = 0, \quad (\text{A.14})$$

where use has been made of the integral identities

$$\int_{-\infty}^{\infty} \delta_0\varphi_0 \exp(-\varphi_0) \frac{d\varphi_0}{d\eta} d\eta = -\frac{1}{2}, \quad \int_{-\infty}^{\infty} \delta_0\varphi_0^2 \exp(-\varphi_0) \frac{d\varphi_0}{d\eta} d\eta = -1$$

and the constant ν is

$$\nu = \int_{-\infty}^{\infty} \delta_0\varphi_0 \exp(-\varphi_0) \frac{d\varphi_0}{d\eta} \eta d\eta = \int_0^{\infty} \left(1 + \frac{d\varphi_0}{d\eta} \right) d\varphi_0 = 1.3440$$

Therefore, the two-term approximation to Δ is found to be given by (42). This approximate solution is shown in Figures 3 and 4 by the dotted lines, which reveal reasonable accuracies in the limit of $m \ll 1$ and $\alpha^- \gg 1$.

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